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# Effect of colour singletness of quark-gluon plasma in quark-hadron phase transition

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**Abstract.** Consequences of the constraint of SU(3) colour singletness of quark-gluon plasma are studied. This restriction increases the free energy barrier for the formation of hadronic bubble in supercooled phase and influences significantly the dynamics of the initial stage of quark-hadron phase transition. It also introduces terms dependent on the volume occupied by the plasma in the energy density and the pressure. These modifactions vanish in the limit of an infinite volume. The last stage of the hadronization of the QGP likely to be formed in relativistic heavy ion collisions is necessarily characterized by a decreasing volume containing the quark matter, and thus these corrections become important. The nucleation of plasma droplets at AGS energies is also seen to be strongly affected by the requirement of colour singletness, and the choice of prefactor.

## 1 Introduction

The success of the quark model, the quantum chromodynamics (QCD), and the non-observability of the free partons ($q$, $\bar{q}$, $g$) has entailed the concept of confinement. QCD, the theory of strong interactions, is not perturbative at large distances. Thus, the confinement itself can not be treated perturbatively. There are reasons to believe that the confinement of partons inside hadrons may not survive collisions between heavy nuclei at relativistic energies. In such collisions, the two nuclei masquerading as clouds of space and time like partons pass through each other, leaving behind a high density plasma of quarks, antiquarks, and gluons (QGP) [1] in their wake, in the region between the two receding fronts of the leading particles. This plasma expands and cools, the energy density becomes low enough and a phase transition to a hadron gas takes place around the critical temperature, $T_C$. A dynamic treatment of this phase transition is a problem of considerable interest.

An understanding of the QCD phase transition requires a knowledge of the equation of state as well as the kinetics of phase transition. If QCD has a first order phase transition, it may proceed with a supercooling of the QGP followed by a nucleation and growth of hadronic bubbles [2,3], releasing the latent heat as the phase transition progresses. In a superheated hadronic matter, on the other hand, nucleation of a QGP droplet may also proceed similarly.

For a first order phase transition the rate for hadronic bubble/plasma droplet nucleation can be estimated in the frame work of homogeneous nucleation theory [4]

$$I = I_0 \exp(-\triangle F^\star/T) \quad , \tag{1}$$

where $I_0$, which has the dimension of $1/\text{fm}^4$ is called the prefactor, $T$ is the temperature, and $\triangle F^\star$ is the change in free energy accompanying the formation of a critical size hadronic bubble/plasma droplet. These dimensional arguments were used in a large number of studies in the past to replace $I_0$ with $T^4$ or $T_C^4$. This unsatisfactory state of affairs was corrected recently by Kapusta and Csernai [5]. They computed the dynamical prefactor in a course-grained effective field theory approximation to QCD. This dynamical factor influences the growth rate and statistical fluctuations and also accounts for the available phase space.

Csernai et al. [2,3] have also used a nucleation rate equation with this realistic dynamical prefactor to study the time evolution of expanding QGP as it converts to hadronic matter. They noted a substantial deviation from an idealized Maxwell construction that has often been employed as a model of hadronization [6]. Obviously, such an idealized phase transition assumes a QCD nucleation rate which is much larger than the rate of expansion. This is not necessarily true.

In all such studies, QGP is generally described as an ideal gas of quarks, antiquarks and gluons, essentially described by the Stefan-Boltzmann law. Lattice calculations [7] have provided ample evidence that even at fairly high temperatures, colour singlet objects like multi-quark cluster ($q\bar{q}$, $qqq$, $\bar{q}\bar{q}\bar{q}$, $\cdots$) propagate in the plasma. One may account for this 'interaction' by requiring that all physical states be colour singlet with respect to the SU(3) colour gauge group [8–11].

It has recently been shown [12] that restricting the quark partition function to be colour singlet of SU(3) colour gauge group amounts to reordering the thermodynamic potential in terms of the colourless multi-quark modes ($q\bar{q}$, $qqq$, $\bar{q}\bar{q}\bar{q}$, $\cdots$) at any given temperature. Under a suitable confining mechanism, these could evolve into colour singlet hadrons/baryons at low temperatures. This is also in accord with the "preconfinement" property of QCD noted by Amati and Veneziano [13] quite sometime ago where the cascading and fragmenting partons produced in hadronic collisions rearrange themselves into colour singlet clusters which ultimately evolve into hadrons [14,15]. These considerations convince us that it is important to incorporate the *dynamic* requirement of colour singletness of the quark-matter which "tunnels" into hadronic matter phase space [15].

In the present work we study the consequences of the incorporation of colour singlet equation of state for plasma on the dynamics of quark-hadron phase transition.

## 2 Equation of state

### 2.1 Colour singlet equation of state for a quark-gluon plasma

Consider a quark-gluon plasma consisting of 'u' and 'd' quarks, and gluons. The grand canonical partition function [9–11] subject to colour singletness can be written as

$$\mathcal{Z}(\beta, V_q) = \mathrm{Tr}\ \left(\hat{\mathcal{P}}\ e^{-\beta \hat{H}}\right)\ , \tag{2}$$

where $\beta = 1/T$ is the inverse temperature, $V_q$ is the volume, $\hat{H}$ is the Hamiltonian of the physical system, and $\hat{\mathcal{P}}$ is the colour projection operator. For a baryon free plasma, this can be simplified after a considerable amount of group theoretic algebraic manipulations [9,11] to give,

$$\mathcal{Z}(\beta, V_q) = \frac{\sqrt{3}}{3\pi}\ \left[\ \frac{8V_q}{3\beta^3}\right]^{-4}\ \exp\ \left[\ \frac{a_q V_q}{\beta^3}\ \right]\ , \tag{3}$$

where $a_q = 37\pi^2/90$. Now the free energy of the baryon-free colourless quark-gluon gas is obtained as,

$$F_q\ =\ -\ T\ \ln \mathcal{Z}(T, V_q)\ +BV_q\ . \tag{4}$$

One may now write for the energy density

$$\begin{aligned} e_q\ &=\frac{E_q}{V_q} \\ &= \frac{T^2}{V_q}\frac{\partial}{\partial T}\left[\ln \mathcal{Z}(\beta, V_q)\right]\ +\ B \\ &= B\ +\ 3a_q T^4\ -\frac{12T}{V_q}\ , \end{aligned} \tag{5}$$

The pressure of the above quark-gluon system is given as

$$\begin{aligned} P_q\ &=\ -\left(\frac{\partial F}{\partial V}\right)_T \\ &= -\ B\ +\ a_q T^4\ -\ \frac{4T}{V_q}\ . \end{aligned} \tag{6}$$

We note that the colour singletness introduces corrections to the normally assumed expresssions for the energy density and the pressure which vanish for an infinite volume. Now consider the hadronic phase arising as a result of first order quark-gluon/hadron transition through the nucleation of hadronic bubble in the bulk QGP. With the passage of time, more and more of the quark matter will get converted to hadronic matter. The later stage of the process of hadronization will be characterized by a decreasing volume occupied by the quark matter. If we believe the above equations of state, the volume occupied by the plasma can not be vanishingly small. The colour singletness will again have important consequences for dynamics of the phase transition during the later stage of hadronization.

Here, we would also like to point out that the colour singletness has important bearing on the nucleation of a hadronic bubble in the plasma which we will see in Sect. 3.

### 2.2 Equation of state for hadron gas

We model the hadronic phase as a gas of massless pions. The energy density and the pressure of such a system can be written as

$$e_h = 3a_h T^4\ , \tag{7}$$

and

$$P_h = a_h T^4\ , \tag{8}$$

where $a_h = \pi^2/30$.

## 3 Supercooling and nucleation

Nucleation in a pure phase like QGP proceeds via creation of a hadronic bubble due to statistical fluctuations in a supercooled plasma. The bubble is made up of hot pion gas and is surrounded by colour singlet plasma of volume, $V_q = (V - V_b)$, where $V$ is initial volume of plasma. $V_b = 4\pi {R_b}^3/3$ represents an excluded volume corresponding to hadronic bubble. One can think of curving out a colour singlet piece of plasma and replacing it with a hadronic bubble. The fields in plasma obey the bag boundary conditions, staying outside the hadronic bubble [16, 17]. If the radius of the bubble is $R_b$, the change in free energy can be written [11,16] within the bag model as,

$$\begin{aligned} \triangle F = T\ln\left(\pi\sqrt{3}\right) + 4T\ln\left(\frac{8}{3}V_b T^3\right) + a_q V_b T^4 \\ -\ (B + P_h)V_b + 4\pi {R_b}^2\sigma\ , \end{aligned} \tag{9}$$

where $P_h$ is the pressure of the hadron gas given in (8) and $\sigma$ is the surface free energy of the quark-gluon/hadron interface. The first two terms in (9) are due to SU(3) colour singlet restriction. They increase the barrier for $\triangle F$ required to form hadronic bubble in plasma (Fig. 1). We shall see later that it has an important effect during the initial stage of QCD phase transition.

Recall that one can derive the critical radius (${R_b}^\star$) of hadronic bubble by minimizing the change in free energy,

$\triangle F$, with respect to $R_b$. If they are too small ($R < R_b{}^\star$), they will shrink and vanish. If they are large ($R > R_b{}^\star$), they will grow. Now, minimizing $\triangle F$ with respect to $R_b$, one gets

$$\frac{12T}{R_b{}^\star} - 4\pi(B - a_{qh}T^4)R_b{}^{\star 2} + 8\pi\sigma R_b{}^\star = 0 \ , \qquad (10)$$

which will yield the critical radius of hadronic bubble $R_b{}^\star$. We have further defined, $a_{qh} = a_q - a_h$. If the first term in (10), which has its origin in the requirement of colour singletness is neglected, one obtains the critical radius, $R_b{}^\star = 2\sigma/(B - a_{qh}T^4)$, for a nonsinglet case. Making a substitution $x = 1/R_b{}^\star$, (10) can be written as

$$x^3 + ax - b = 0 \ , \qquad (11)$$

where $a = 2\pi\sigma/3T$ and $b = \pi(B - a_{qh}T^4)/3T$. The physical solution of (11) gives the critical radius of hadronic bubble for the colour singlet case, as,

$$R_b{}^\star = 3z/\left(3z^2 - a^2\right); \quad z^3 = \left[b/2 + \sqrt{a^3/27 + b^2/4}\right] \ . \qquad (12)$$

Now, the change in free energy for the creation of a hadronic bubble having the critical radius $R_b{}^\star$ is

$$\triangle F_\star = \triangle F|_{R=R_b{}^\star} \ . \qquad (13)$$

The rate for the nucleation of hadronic phase out of the plasma phase is usually estimated by (1) with prefactor, $I_0$. As remarked earlier the prefactor, $I_0$, has been calculated by Csernai and Kapusta [5] in an effective field theory approximation to QCD as

$$I_0 \ = \ \frac{16}{3\pi}\left(\frac{\sigma}{3T}\right)^{3/2}\frac{\sigma\eta_q R_b{}^\star}{\xi_q^4(\triangle w)^2} \ . \qquad (14)$$

Here, $\eta_q$ and $\xi_q$ are, respectively, the shear viscosity and correlation length in the plasma phase, and $\triangle w$ is the difference in the enthalpy densities of the two phases. We use the same parameter set as used in [2,3], e.g., $B^{1/4} =$ 235 MeV, $\xi_q = 0.7$ fm, $\eta_q = 14.4T^3$ and $T_C = 169$ MeV. Next we closely follow the arguments of [2,3] to obtain the dynamics of the phase transition.

Once the nucleation rate is known, one can calculate the (volume) fraction of space $h(\tau)$ converted to hadronic gas at a proper time $\tau$. This proper time is measured in the local comoving frame of an expanding system. For this purpose one needs a kinetic equation which involves $I$ as the source. If the system cools to $T_C$ at a time $\tau_C$, then at some later time $\tau$ the fraction of the space which has converted to hadronic gas is [2,3]

$$h(\tau) \ = \ \int_{\tau_C}^{\tau} \mathrm{d}\tau' I\left(T(\tau')\right)\left[1 - h(\tau')\right]V(\tau',\tau) \ , \qquad (15)$$

where, $V(\tau',\tau)$ is the volume of a bubble at a time $\tau$ which has nucleated at an earlier time $\tau'$. This also takes into account bubble growth. How rapidly does the bubble grow after nucleation? Usually a critical size bubble is metastable and will not grow without a perturbation. Pantano and Miller [18] have numerically computed the growth of bubbles using a relativistic hydrodynamics. The asymptotic radial growth velocity was found to be consistent with the growth law

$$v(T) \ = \ v_0\left(1 - T/T_C\right)^{3/2} \ , \qquad (16)$$

where $v_0$ is a model-dependent constant. We shall use $v_0 =$ $3c$ as it has been argued [2,3] that the (16) is intended to be applied only as long as $T > 2T_C/3$ so that the growth velocity stays below the speed of sound of a massless gas, $c/\sqrt{3}$. Thus the growth of the bubble can be approximated as,

$$V(\tau',\tau) \ = \ \frac{4}{3}\pi\left[R_b{}^\star\left(T(\tau')\right) \ + \ \int_{\tau'}^{\tau}\mathrm{d}\tau'' v\left(T(\tau'')\right)\right]^3 \ . \qquad (17)$$

Now one needs a dynamical equation which couples the time evolution of the temperature to the fraction of space converted to hadronic gas. For this purpose we use both Bjorken longitudinal hydrodynamics and Cooper-Frye-Schönberg spherical hydrodynamics as [2,3]. In Bjorken model [19] the time evolution of energy density $e$ is given as

$$\frac{\mathrm{d}e}{\mathrm{d}\tau} \ = \ -\frac{w}{\tau} \ , \qquad (18)$$

whereas in Cooper-Frye-Schönberg model [20] it is

$$\frac{\mathrm{d}e}{\mathrm{d}\tau} \ = \ -\frac{3w}{\tau} \ . \qquad (19)$$

In (19) the factor of 3 appears because of the spatial expansion along three dimensions rather than one dimension. In order for an easy comparison we consider initial conditions as in [2,3] which are likely to be achieved in collisions involving two gold nuclei at RHIC energies. Thus for longitudinal expansion we take $\tau_i = 3/8$ fm/c and $T_i = 2T_C$. The temperature decreases as $T(\tau) \propto \tau^{-1/3}$ until the time $\tau_C = 3$ fm/c when the temperature becomes $T_C$. In the case of spherical expansion we consider $T_i = 2T_C$ and $\tau_i = \sqrt{2}R_{\rm nucl}$ and the temperature decreases like $T(\tau) \propto \tau^{-1}$ until the time $\tau_C = 18$ fm/c corresponding to half density radius of a gold nucleus (For details see [2,3]). The equations (18) and (19) are essentially the statement of energy conservation in the respective picture which assume kinetic equilibrium but not phase equilibrium. The energy density can be written [2,3] as

$$e(T) \ = \ h(\tau)e_h(T) \ + \ \left[1 - h(\tau)\right]e_q(T) \ . \qquad (20)$$

Here, $e_h$ and $e_q$ are the the energy densities in hadronic and plasma phases at temperature $T$, and similarly for $w$.

## 4 Dynamics of phase transition: Longitudinal and spherical expansion

In Fig. 1 the free energy difference, $\triangle F$ in (9) as a function of hadronic bubble radius for a fixed bag constant

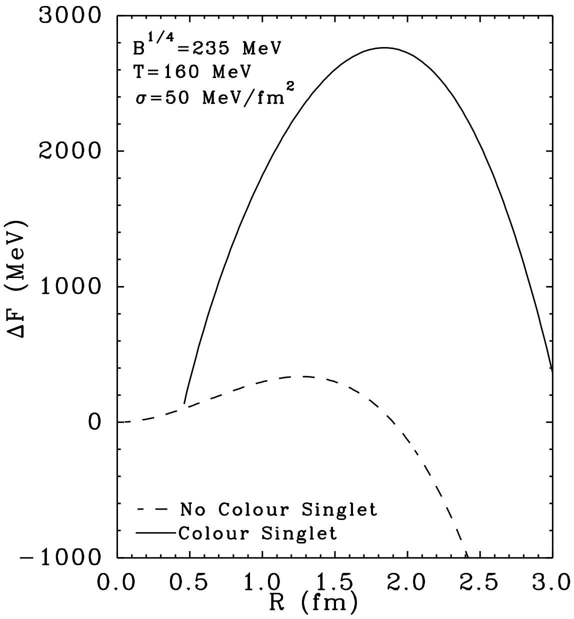


**Fig. 1.** The free energy difference $\triangle F(R)$ for creation of a hadronic bubble in quark-gluon plasma

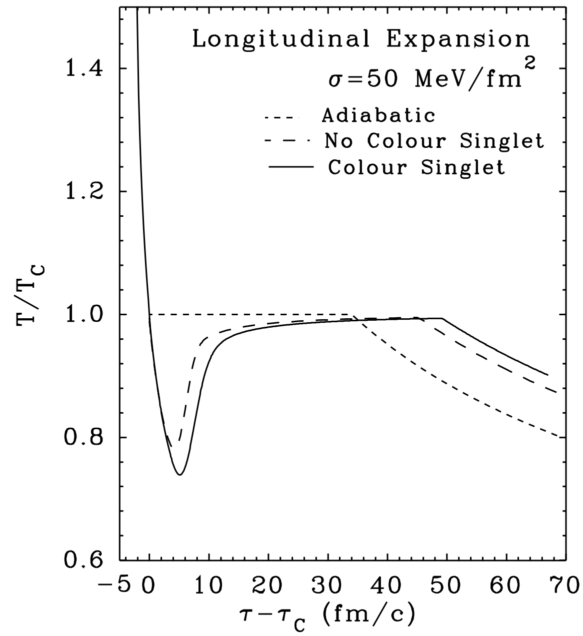


**Fig. 2.** The temperature as function of proper time for a hadronizing quark-gluon plasma in a central high-energy nucleus-nucleus collision for matter undergoing longitudinal expansion. The initial conditions correspond to QGP formation at BNL - RHIC energies

($B^{1/4} = 235$ MeV), surface tension ($\sigma = 50$ MeV/fm$^2$) and temperature ($T = 160$ MeV) is displayed. Results are given with and without colour singlet requirement of the system. One finds a significant increase in $\triangle F$ if the constraint of colour singletness is imposed. We will see below that it has important consequences on the time evolution of expanding QGP as it converts to hadronic matter through nucleation of hadronic bubble.

In Fig. 2 we show the variation of the temperature with proper time as the matter undergoes longitudinal expansion with the initial conditions given in Sect. 3. We have also given the results for an adiabatic phase transition for a comparision. The value of $\sigma$ used here is 50 MeV/fm$^2$. The general features are similar to those of [2,3].

The matter continues to cool below $T_C$ until nucleation of hadronic bubble sets in. For colour singlet case the degree of supercooling is about 30%, *i.e.*, 12.5% more than that for the nonsinglet case. Once nucleation and growth of bubble start, the system reheats near $T_C$ due to release of latent as the phase transition progresses. When temperature approaches $\sim 0.95 T_C$ nucleation of further bubble formation ceases off and the transition proceeds because of growth of previously nucleated bubbles. However, the system can not reheat upto $T_C$ because bubble growth (17) reduces to zero as $T_C$ is approached. Recall that nucleation in colour singlet case will be delayed due to increase in height of $\triangle F$ which in turn also slows down the phase transition. This results in 10% extra entropy generation in the processs as compared to colour nonsinglet case.

Figure 3 shows the variation of critical radius as a function of proper time for hadronic bubble undergoing longitudinal expansion with and without colour singlet requirement. It clearly indicates that the early stage of nucleation is characterized by a much larger critical size of nucleated hadronic bubble ($\sim 1$ fm) for colour singlet case as compared to $\sim 0.5$ fm for colour nonsinglet case. At later times ($\geq 12$ fm/c) the critical size of nucleated hadronic bubble remains lower until the phase transition is completed if the colour singlet requirement is implemented in the system. In these initial studies we have not included the modification due to bubble fusion which should be of interest.

In Fig. 4 we show the volume fraction converted to hadronic matter undergoing longitudinal expansion as a function of proper time. The delay in completion of QCD phase transition due to colour singletness is seen clearly. As the phase transition progresses via nucleation of hadronic bubble, the available space will be progressively occupied by hadronic matter. We would like to make an amusing observation here. If we believe the equations of state for the QGP as discussed earlier ((5) and (6)), the volume occupied by plasma can *not* be vanishingly small. This necessarily implies a remnant of quark matter when the process of hadronization is over. We find that a fraction of quark matter of mass $\sim$ (5-10) GeV having volume $\sim$ 10–15 fm$^3$ remains unconverted at the end of the hadronization in the central region, per unit rapidity. One can have interesting speculations about such a remnant.

In Fig. 5 we have attempted to study the dynamics of phase transititon as a function of quark/hadron interface tension $\sigma$. Recall [3] that the dynamics depends sensitively on $\sigma$ when we do not impose the restriction of colour singletness. With imposition of colour singletness large variation in $\sigma$ leaves the dynamics fairly unchanged. The reason for this is not too difficult to identify. Note that (10) tells us that even if $\sigma = 0$, there is

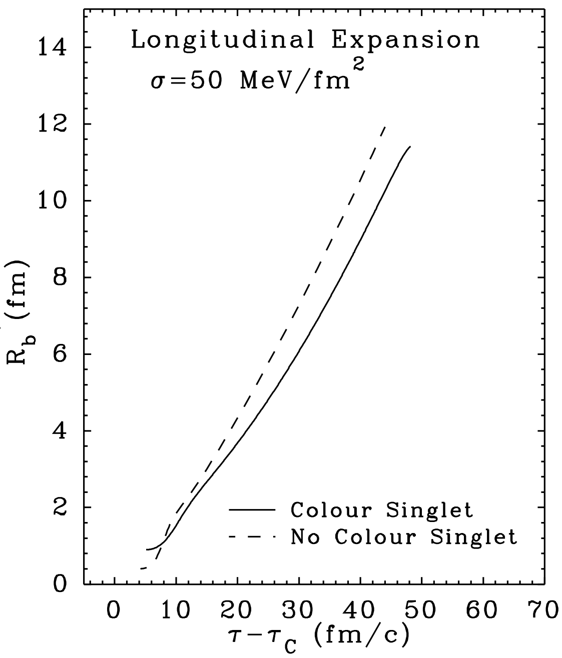


**Fig. 3.** Radius of critical hadronic bubbles as a function of time, in a hadronizing quark-gluon plasma

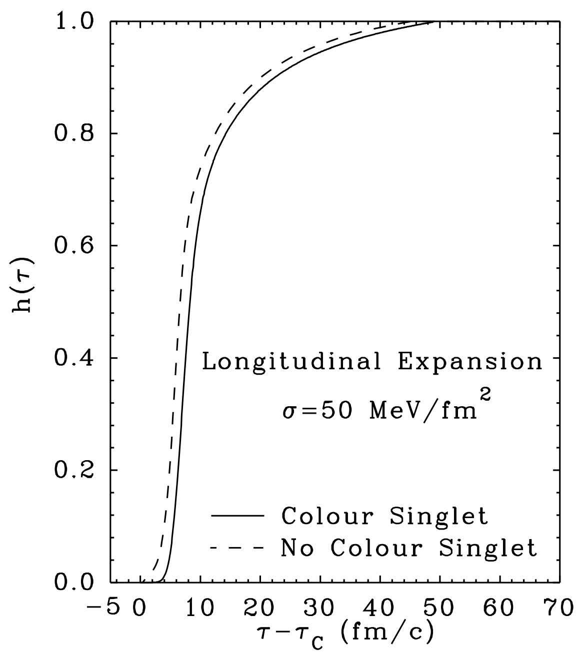


**Fig. 4.** The volume-fraction of space occupied by the hadronic matter as a function of time, in a hadronizing quark-gluon plasma

possibility of nucleation of hadronic bubble with a critical radius, $R_b{}^\star = [3T/\pi(B - a_{qh})]^{1/3}$. However as the nucleation rate, $I$, itself depends on $\sigma$ via its prefactor, $I_0$ and $I = 0$ if $\sigma = 0$. Thus we have kept $\sigma = 50$ MeV/fm$^2$ as $I$ involves a more realistic prefactor characterized by $\sigma$.

How will these findings differ for a more realistic (3+1) dimensional expansion of plasma? Normally, the plasma is expected to expand mostly in longitudinal direction ini-

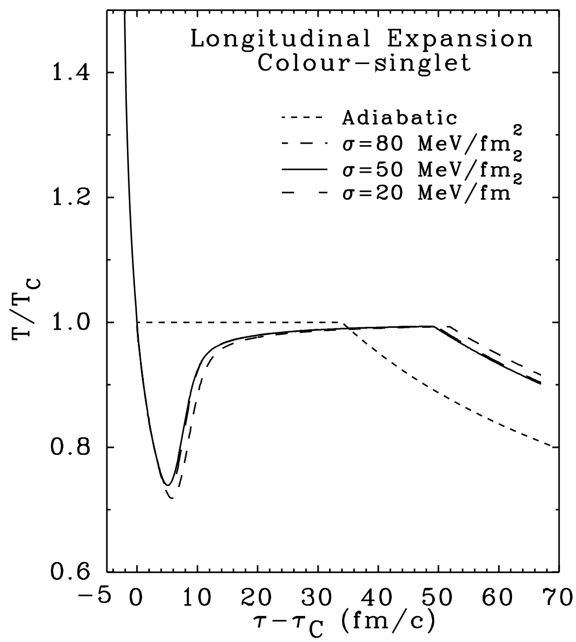


**Fig. 5.** Same as Fig. 2 with various $\sigma$ values for colour singlet case

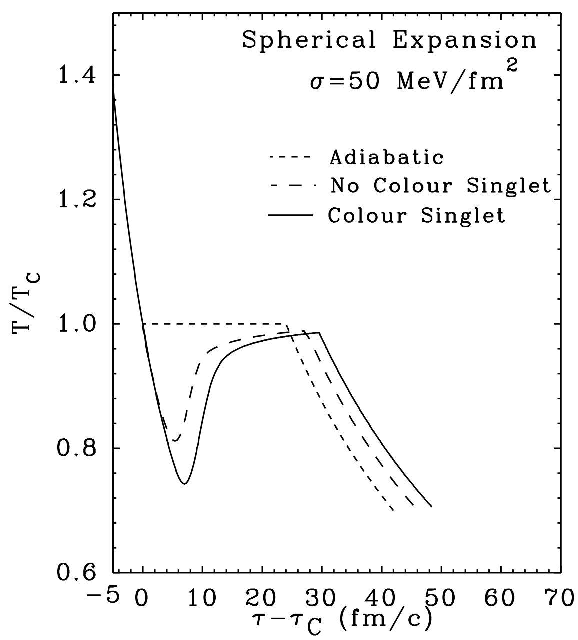


**Fig. 6.** Same as Fig. 2 for matter undergoing spherical expansion

tially. After a time $\tau \simeq R/c_s$ where R is transverse radius and $c_s$ is the speed of sound, the system is likely to expand in transverse direction as well. We, like the authors of [3], take the other extreme and look on spherical expansion with the same initial conditions as in [3].

Figure 6 is similar to Fig. 2 with the exception that it considers spherical (3-dimensional) expansion instead of longitudinal (1-dimensional) expansion. The degree of supercooling is almost of the order of longitudinal one (30%) with the prime difference that hadronization is faster in

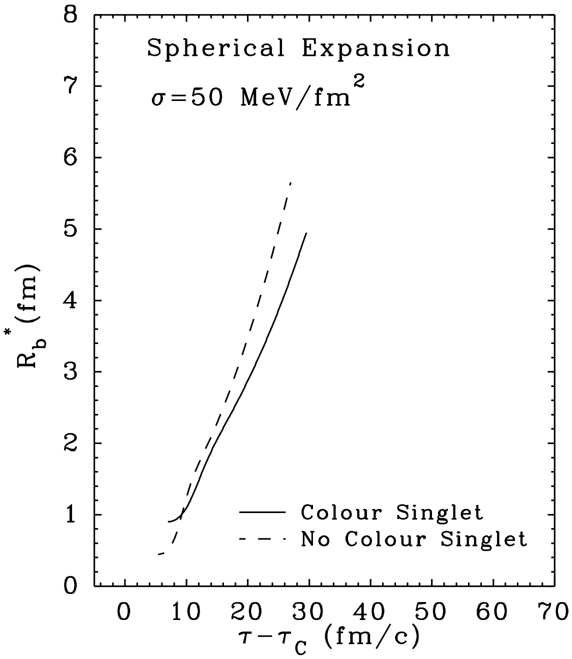


**Fig. 7.** Same as Fig. 3 for matter undergoing spherical expansion

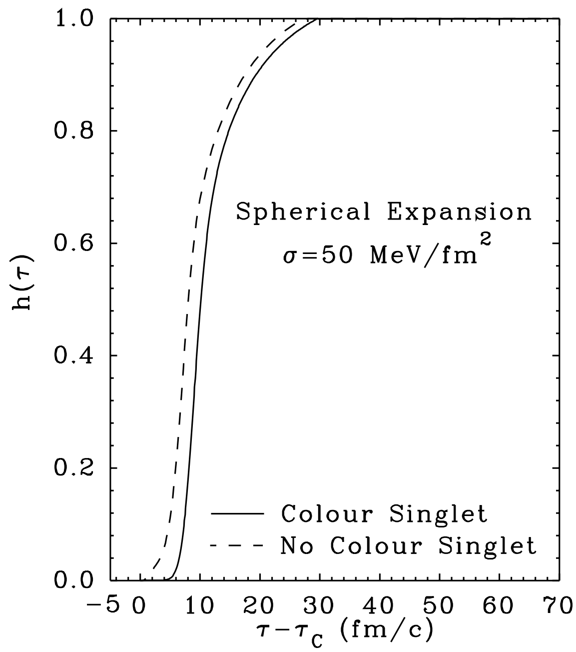


**Fig. 8.** Same as Fig. 4 for matter undergoing spherical expansion

3-dimensional case. This implies that the system spends less time in the neighbourhood of $T_C$ than otherwise. Figure 7 shows the variation of critical radius of hadronic bubbles undergoing spherical expansion as a function of proper time with and without colour singletness. It is clear that the critical radius of nucleated hadronic bubbles for spherical expansion is smaller compared to longitidunal expansion. In Fig. 8 we show the variation of volume fraction converted to hadronic matter for spherical expansion as a function of proper time. This is also clear from this figure that the phase transition is faster than the longitudinal one. For the shake of completeness we also present Fig. 9 for spherical expansion which shows the variation of temperature as a function of proper time for different quark/hadron interface values.

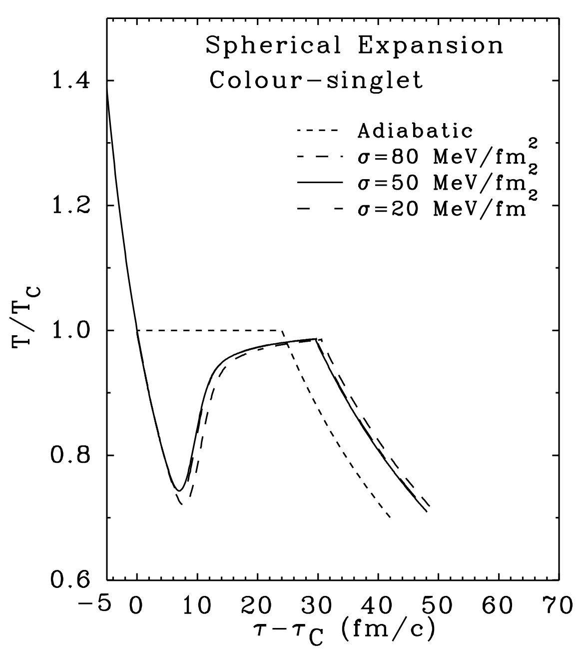


**Fig. 9.** Same as Fig. 5 for matter undergoing spherical expansion

## 5 Nucleation rate of droplets of quark-gluon plasma in hot hadron gas

The creation of QGP at AGS energies may proceed through the nucleation of a plasma droplet in hot hadronic gas. Here the fields in plasma obey the bag boundary conditions, staying inside the plasma droplet [16]. If the radius of the droplet is $R_q$, the nucleation process is, generally, activated by the change in free energy which can be written within the bag model [11,16] as

$$\triangle F = T\ln\left(\pi\sqrt{3}\right) + 4T\ln\left(\frac{8}{3}V_qT^3\right) - a_qV_qT^4 + (B+P_h)\,V_q + 4\pi {R_q}^2\sigma \ . \qquad (21)$$

Here, $V_q$ is volume of the plasma droplet formed in a superheated hadron gas. Usually, the nucleation rate of a plasma droplet from superheated hadronic phase is estimated from (1). Now, the prefactor [5] is given by

$$I_0 \ = \ \frac{\kappa}{2\pi}\Omega_0 \ . \qquad (22)$$

The dynamical prefactor $\kappa$, determines the exponential growth rate of critical size droplets, and is given by [21]

$$\kappa \ = \ \frac{2\sigma}{(\triangle w)^2{R_q}^{\star 3}}\ \left[\lambda T + 2\left(\frac{4}{3}\eta+\zeta\right)\right] \ , \qquad (23)$$

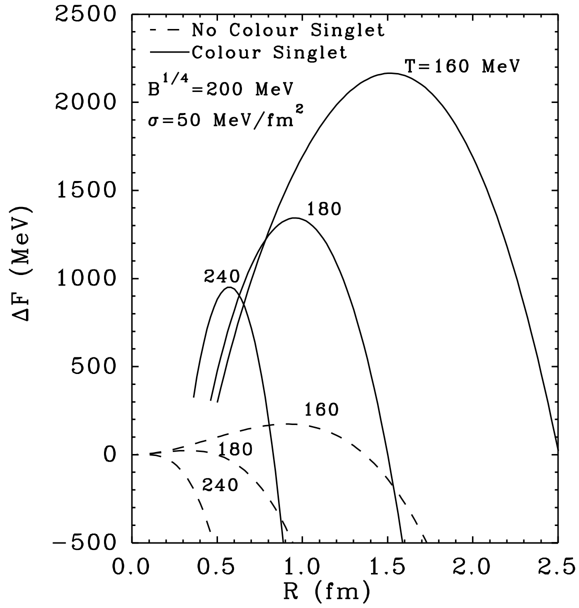


**Fig. 10.** The free energy difference $\triangle F(R)$ for creation of a plasma droplet in hot hadronic matter

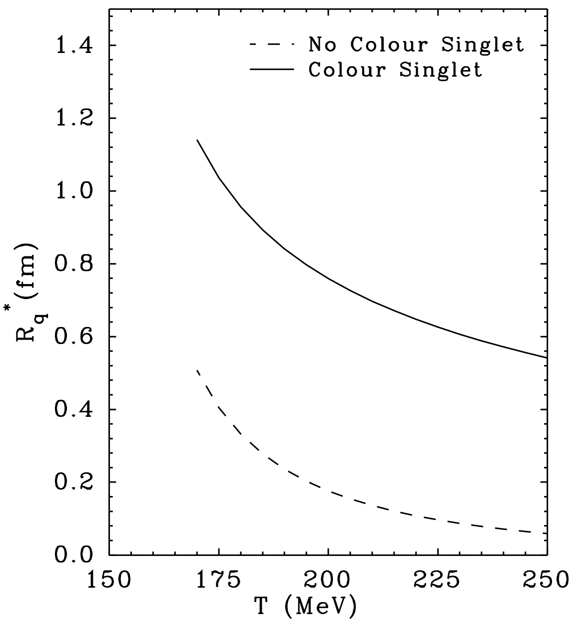


**Fig. 11.** Critical radius of nucleated plasma droplets as a function of temperature in a superheated hadronic matter

where $\lambda$ is the thermal conductivity and $\eta$ and $\zeta$ are the viscosities of the hadronic phase. The bulk viscosity $\zeta$ is very small compared to shear viscosity $\eta$ and can be neglected. For these dissipative coefficients we use the parameterization of Danielewicz [22]. $R_q{}^\star$ is the critical radius of a nucleated plasma droplet. This can be obtained by minimizing $\triangle F$, given in (21), with respect to $R_q$. To a first approximation the statistical prefactor [5, 21] is given by

$$\Omega_0 \; = \; \frac{2}{3\sqrt{3}} \left(\frac{\sigma}{T}\right)^{3/2} \left(\frac{R_q{}^\star}{\xi_h}\right)^4 . \qquad (24)$$

For our purpose we use following set of parameter $\sigma = 50$ MeV/fm$^2$, $\xi_h = 0.7$ fm and $B^{1/4} = 200$ MeV which gives $T_C \sim 170$ MeV. We shall also give results for the prefactor $\sim T^4$ which is used often in such studies.

As a first step, in Fig. 10 we plot the variation of $\triangle F$ as a function of droplet radius with and without colour singlet restriction for three different temperatures (160, 180, 240 MeV). Due to colour singlet restriction, the height of $\triangle F$ is enhanced significantly in each case. Figure 11 shows the variation of critical radius of nucleated plasma droplet as a function of temperature. We see that the imposition of the colour singletness increases the critical radius by factor of $\sim$ 3–5 over the range of temperatures that we have considered here. Knowing that creating a bigger size bubble by statistical fluctuation is considerably less probable, it is not surprising that this should lead to considerable suppression of nucleation rate when the degree of superheating is small.

Figure 12 shows the variation of nucleation rate of plasma droplets in superheated hadron gas as a function of temperature. It is seen that the nucleation rate with the prefactor $T^4$ (dashed lines) is suppressed considerably when the restriction of colour singletness is imposed. Similar finding has been reported recently in the literature [23] with a somewhat different expression for $\triangle F$. The results (solid lines) with the more realistic prefactor (22) are richer in detail. Firstly we notice that without the restriction of colour singletness the traditional prefactor considerably overestimates the rate of nucleation. With prefactor given in (22), we see that switching on the requirement of colour singletness lowers the rate of nucleation at smaller $T$. However, in a surprising finding, we note that while the no-colour-singlet rate decreases with increase of $T$, that with colour singlet restriction increases and ultimately becomes larger at really high $T$. This interesting behaviour is seen to emerge from the structure of the prefactor (22) which in fact is proportional to $R_q{}^\star$, and $R_q{}^\star$ is seen to decrease with increase in $T$ (Fig. 11). We add here that Kapusta and Vischer [24] have recently proposed a more efficient mechanism for nucleation of plasma droplet at AGS energies which envisages seeding of the plasma via collision of two very energetic nucleons in the hot and dense hadronic matter.

## 6 Summary

Before summarizing let us briefly examine some of the inputs in the present work. While writing the expressions for the gain/loss in free energy ($\triangle F$) we have included only the volume and the surface terms. This should be valid when the size of the plasma droplets or hadronic bubbles is large. The prefacor we have used is valid for the situation when the size of the bubbles/droplets is larger than the correlation length of the system ($\xi = 0.7$ fm). We have seen that these bubbles/droplets have radii larger than a fermi, and thus both these conditions are reasonably satisfies.

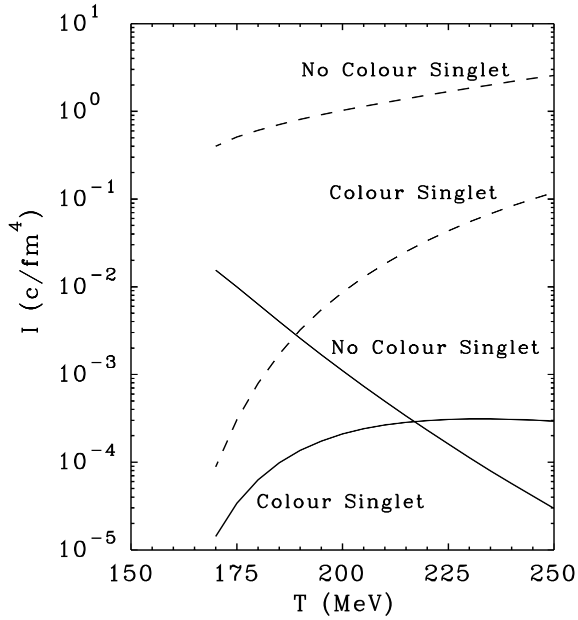


**Fig. 12.** The nucleation rate of plasma droplets as a function of temperature, in a superheated hadronic matter. The *solid lines* are with dynamical prefactor of [5] whereas those with *dashed lines* are with $T^4$ as prefactor

Recall that the surface energy coefficient $\sigma$ is zero in the MIT bag model for massless quarks [25]. Quarks traversing a hot medium, do acquire a thermal mass. Finite temperature lattice QCD [26] and a pure SU(N) gauge theory [27] yield a value in the range $\sigma \approx$ 20–70 MeV/fm$^2$. We have thus mostly used $\sigma \approx 50$ MeV/fm$^2$ and the effect of varying $\sigma$ is also studied. We have not come across any estimate of the so-called curvature term in this case. We have verified, however, if we add a term for curvature energy in the expression of $\triangle F$ for the creation of hadronic bubble in the plasma as in [16], then the supercooling of the plasma is completely eleminated, if we ignore the colour singletness. We have also checked that supercooling of plasma is possible if the colour singlet restriction is imposed along with curvature contribution. However, for plasma droplet in hot hadronic gas the degree of superheating will be enhanced than otherwise.

In brief, the theory, proposed recently [2], to describe the dynamics of hadronization has been generalized to study the consequences of the requirement of colour singletness of QGP which may be produced in relativistic heavy ion collisions. It is shown that hadronization of longitudinally and spherically expanding plasma may be slowed down due to this requirement. While there is no production of entropy for an adiabatic phase transition, the entropy increases by about 30% in the treatment of [2]. The requirement of colour-singletness introduced in the present work enhances this entropy by an additional 10% and also increases the degree of supercooling. We note an interesting possibility; a small fraction of QGP may not hadronize at all.

We also find that the nucleation of a plasma droplet in a superheated hadronic matter is suppressed at low temperature ($T \leq 220$ MeV), but it is enhanced at higher temperature ($T \geq 220$ MeV) when the colour singlet restriction of QGP is accounted for. The decrease at $T \sim 170$ MeV is found to be by *four orders of magnitude* while the enhancement at 250 MeV is by one order of magnitude.

It would be of interest to consider a generalization of this study to the case of non-equilibrated plasma which hadronizes into a hadronic matter having a richer equation of state. A (rather) crude result may be obtained by keeping all other parameters like $T_C$, $\sigma$, $\eta$, etc. fixed to their present values. This we feel, may not be quite justified.

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